section 4–m) is shown in Fig. 4b (line 1 for $\Theta = 0.36$ and line 2 for $\Theta = 0.18$). The location of the separation line for $\Theta_w = 0.18$, $\Theta_t = 0.36$, and $\lambda = 1$ is denoted by 5 in Fig. 1 (the solid line shows the direction of the flow), and for $\Theta_w = 0.18$, $\Theta_t = 0.36$, and $\lambda = 2.16$ by 6 (the dashed line shows the direction of the flow). The influence of the flow nonisothermicity on the position of the separation line is not essential.

The numerical experiment conducted to determine the effect of the distance between the fictitious boundaries on the flow pattern showed that, while this distance decreases by δ, the separation line is displaced by approximately $\delta/2$ in the direction which provides better agreement with the experimental data. The influence of the position of the fictitious boundaries on the calculated position of the separation line was also examined. It was established that the calculated position of the separation line is not affected by changes of the position of the fictitious boundaries where the proper velocity profile was assigned. The position of the fictitious boundaries, chosen as hk and op in Fig. 1, seems to be more suitable for investigating the influence of nonisothermicity on the flow pattern, and more convenient from the point of view of numerical realization.

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ACOUSTIC PROPERTIES OF A POROUS LAMINATED MEDIUM

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Study of the propagation of elastic waves in nonuniform, saturated, porous media is of interest both theoretically and from the viewpoint of applications in engineering and geophysics. The propagation of elastic waves in such media can be systematically described within the framework of the Frenkel–Biot theory [1-4]. Ignoring thermoelastic effects, we can write the equations of this theory in the form

$$\rho_{11}\frac{\partial^2 u_i}{\partial t^2} + \rho_{12}\frac{\partial^2 v_i}{\partial t^2} = b\frac{\partial}{\partial t}(v_i - u_i) - \frac{\partial P_{ij}}{\partial x_j}, \tag{1}$$

$$\rho_{12}\frac{\partial^2 u_i}{\partial t^2} + \rho_{22}\frac{\partial^2 v_i}{\partial t^2} = b\frac{\partial}{\partial t}(u_i - v_i) - \frac{\partial s}{\partial x_i},$$

where u_i and v_i are components of the displacement vectors of the skeleton and fluid; ρ_{11} is the effective density of the skeleton moving in the filler; ρ_{22} is the effective density of the filler moving in the porous medium; $\rho_{12} < 0$ is the added density of the fluid; $P_{ij} = Ae\delta_{ij} + 2Ne_{ij} + Q\varepsilon\delta_{ij}$; $s = Qe + R\varepsilon$; $e = \mathrm{div}\,\mathbf{u}$; $\varepsilon = \mathrm{div}\,\mathbf{v}$; $e_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$; A, N, Q, R are constants of the elastic constraints of the porous medium; the coefficient b characterizes friction due to the motion of the fluid: $b = \mu\Phi^2/K_{pr}$ (μ is the viscosity of the fluid, Φ is bulk porosity, and K_{pr} is the permeability).

After we introduce the two scalar potentials φ_1 and φ_2 of the longitudinal waves and the vector potential of the transverse wave by means of the relations

$$\mathbf{u} = \nabla\varphi_1 + \nabla\varphi_2 + \mathrm{rot}\boldsymbol{\Psi}; \tag{2}$$

$$\mathbf{v} = M_1\nabla\varphi_1 + M_2\nabla\varphi_2 + M_t\mathrm{rot}\boldsymbol{\Psi} \tag{3}$$

in the case of harmonic waves, system (1) reduces to a system of two scalar Helmholtz equations

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$$\Delta\varphi_i + k_i^2\varphi_i = 0 \quad (i = 1, 2) \tag{4}$$

and the vector equation

$$\Delta\Psi + k_t^2\Psi = 0. \tag{5}$$

Here, $M_i = (\gamma_{12} - \xi_i\sigma_{12} + i\gamma)/(-\gamma_{22} + \xi_i\sigma_{22} + i\gamma)$ $(i = 1, 2)$; $k_i^2 = \xi_i(\omega/v_0)^2$; $v_0 = \sqrt{H/\rho}$; $H = A + 2N + 2Q + R$; $\rho = \rho_{11} + 2\rho_{12} + \rho_{22}$, while ξ_i are roots of the quadratic equation

$$\xi_i^2(\sigma_{11}\sigma_{22} - \sigma_{12}^2) - \xi_i(\sigma_{11}\gamma_{22} + \sigma_{22}\gamma_{11} - 2\sigma_{12}\gamma_{12} - i\gamma) + \gamma_{11}\gamma_{22} - \gamma_{12}^2 - i\gamma = 0, \tag{6}$$

in which $\gamma_{11} = \rho_{11}/\rho$; $\gamma_{12} = \rho_{12}/\rho$; $\gamma_{22} = \rho_{22}/\rho$; $\gamma = -b/\rho\omega$; $\sigma_{11} = (A + 2N)/H$; $\sigma_{12} = Q/H$; $\sigma_{22} = R/H$. The wave number of the transverse wave, k_t, is found from the relation $k_t^2 = \omega^2\rho/N[\gamma_{11} + \gamma_{12} + i\gamma(M_t - 1)]$ $(M_t = -(\gamma_{12} + i\gamma)/(\gamma_{22} - i\gamma))$.

Dispersion relation (6) has two different roots. These roots determine the wave numbers of longitudinal waves of the first and second types [2, 3], which propagate independently in a uniform medium. The first type of longitudinal wave approximately corresponds to cophasal motion of the skeleton and fluid. It propagates rapidly with little attenuation. In contrast to this, the skeleton and fluid undergo antiphase motion in a longitudinal wave of the second type. Thus, the attenuation of this wave is usually great, and special experiments are required to record it. Formation of longitudinal waves of the second type near interfaces leads to dissipation of the energy of elastic vibrations and a change in the characteristics of reflected and refracted longitudinal waves of type one and transverse waves [3].

The Thomson–Haskell matrix method [6, 7] was used in [5] to perform calculations of the reflection and transmission coefficients for a seismic wave moving through a system containing n different porous strata.

Here, we calculate the effective wave numbers of elastic waves propagating in a porous medium consisting of an infinite structure of periodically alternating layers with different properties. The problem of the propagation of elastic plane waves along or across bedding was solved in [8] for a one-phase elastic medium. The results in [8] were later generalized in [9], where a study was made of the propagation of plane quasi-longitudinal and quasi-transverse waves in multilayered periodic structures. We will use the formalism developed in [9] to study the acoustic properties of a periodically layered porous medium.

We will examine a two-dimensional structure, the properties of which change periodically along the z axis. The properties are constant along the x axis. Wave propagation in each layer will be described by Eqs. (4), (5), the solution of which has the form

$$\varphi_i^{(m)} = B_{i1}^{(m)} \exp\left\{i\left[\xi x + \alpha_i^{(m)}\left(z - \frac{h_m + h_{m+1}}{2}\right)\right]\right\} + \tag{7}$$

$$+ B_{i2}^{(m)} \exp\left\{i\left[\xi x - \alpha_i^{(m)}\left(z - \frac{h_m + h_{m+1}}{2}\right)\right]\right\} \quad (i = 1, 2),$$

$$\Psi = \Psi\mathbf{n}_y = C_1^{(m)} \exp\left\{i\left[\xi x + \beta_m\left(z - \frac{h_m + h_{m+1}}{2}\right)\right]\right\}\mathbf{n}_y +$$

$$+ C_2^{(m)} \exp\left\{i\left[\xi x - \beta_m\left(z - \frac{h_m + h_{m+1}}{2}\right)\right]\right\}\mathbf{n}_y,$$

where m and h_m are the number and thickness of the layer; $\mathbf{n}_y$ is a unit vector in the y direction; $B_{ik}^{(m)}$, $C_k^{(m)}$ are constants determined from the boundary conditions; ξ is the component of the wave vector in the direction x; $\tilde{\alpha}_i^{(m)} = \sqrt{k_i^{(m)2} - \xi^2}$; $\beta_m = \sqrt{k_t^{(m)2} - \xi^2}$; the multiplier $\exp(-i\omega t)$ is omitted for the sake of brevity.

The following boundary conditions should be satisfied at the boundaries of the layers

$$\Gamma_{nn} = \Gamma'_{nn}, \quad \Phi's = \Phi s', \quad \Gamma_{n\tau} = \Gamma'_{n\tau}, \quad u_n = u'_n, \tag{8}$$

$$(1 - \Phi)u_n + \Phi v_n = (1 - \Phi')u'_n + \Phi' v'_n, \quad u_\tau = u'_\tau.$$

Here, Γ_{nn} and $\Gamma_{n\tau}$ are the normal and tangential components of the tensor of the total stresses, which is connected with the tensor of the Biot stresses P_{ij} by the relation $\Gamma_{ij} = P_{ij} + \delta_{ij}s$; the first and third conditions express the continuity of the normal and tangential components of the stress tensor, the second describes the equality of pressures in the fluid at the interface, the fourth and fifth express the continuity of the normal displacements of the skeleton and the total displacements, respectively, and the sixth describes the continuity of the tangential displacement of the skeleton at the interface.

The following matrix equality, a consequence of boundary conditions (8), exists at the interface of the m-th and (m − 1)-st layers:

$$D_{m-1}Z_{m-1}X_{m-1} = D_m Z_m^{-1} X_m, \tag{9}$$

where X_m is a vector containing constants introduced by Eq. (7); Z_m is a diagonal matrix, the nonvanishing elements of which $Z_m = \| \exp(i\alpha_1^{(m)} h_m/2),\ \exp(-i\alpha_1^{(m)} h_m/2),\ \exp(i\alpha_2^{(m)} h_m/2),\ \exp(-i\alpha_2^{(m)} h_m/2),\ \exp(i\beta_m h_m/2),\ \exp(-i\beta_m h_m/2) \|$, while the elements of the matrix D_m have the form

$$D_{m11} = 2N^{(m)}\xi^2 - k_1^{(m)2}\left[A^{(m)} + 2N^{(m)} + Q^{(m)} + M_1^{(m)}\left(Q^{(m)} + R^{(m)}\right)\right],\ D_{m12} = D_{m11},\ D_{m13} = 2N^{(m)}\xi^2 - k_2^{(m)2}\left[A^{(m)} + 2N^{(m)} + Q^{(m)} + M_2^{(m)}\left(Q^{(m)} + R^{(m)}\right)\right],$$

$$D_{m14} = D_{m13},\ D_{m15} = -2N^{(m)}\xi\beta_m,\ D_{m16} = -D_{m15},$$

$$D_{m21} = -k_1^{(m)2}\left(Q^{(m)} + M_1^{(m)}R\right)/\Phi_m,\ D_{m22} = D_{m21},\ D_{m23} = -k_2^{(m)2}\left(Q^{(m)} + M_2^{(m)}R\right)/\Phi_m,\ D_{m24} = D_{m23},\ D_{m25} = 0,\ D_{m26} = 0,\ D_{m31} = -2N^{(m)}\xi\alpha_1^{(m)},$$

$$D_{m32} = 2N^{(m)}\xi\alpha_1^{(m)},\quad D_{m33} = -2N^{(m)}\xi\alpha_2^{(m)},\ D_{m34} = 2N^{(m)}\xi\alpha_2^{(m)},$$

$$D_{m35} = N^{(m)}\left(2\beta_m^2 - k_t^{(m)2}\right),\ D_{m36} = N^{(m)}\left(2\beta_m - k_t^{(m)2}\right),$$

$$D_{m41} = D_{m42} = D_{m43} = D_{m44} = \xi,\ D_{m45} = -\beta_m,\ D_{m46} = \beta_m,$$

$$D_{m51} = -D_{m52} = \alpha_1^{(m)},\ D_{m53} = -D_{m54} = \alpha_2^{(m)},\ D_{m55} = \xi,\ D_{m56} = \xi,$$

$$D_{m61} = \alpha_1^{(m)}\left(1 - \Phi_m + M_1^{(m)}\Phi_m\right),\ D_{m62} = -D_{m61},\ D_{m63} = \alpha_2^{(m)}\left(1 - \Phi_m + M_2^{(m)}\Phi_m\right),$$

$$D_{m64} = -D_{m63},\ D_{m65} = \xi\left(1 - \Phi_m + M_t^{(m)}\Phi_m\right),\ D_{m66} = \xi\left(1 - \Phi_m + M_t^{(m)}\Phi_m\right).$$

Successively applying Eq. (9) to the interfaces of different layers, we have the matrix equality $X_m = Z_m D_m^{-1} D_{m-1} Z_{m-1}^2 D_{m-1}^{-1} \ldots D_0 Z_0 X_0$. Following [6], we can easily use this equality to obtain a dispersion relation for the waves in a periodically layered porous medium:

$$|\exp(iLa)E - T_{m-1}T_{m-2} \ldots T_0| = 0, \tag{10}$$

where E is a unit matrix; L is the period of the structure; a is the sought effective wave number; $T_m = D_m Z_m^2 D_m^{-1}$.

In contrast to the dispersion relations for a periodically laminated elastic medium, (10) has not four, but six independent roots. These roots correspond to the propagation of two quasi-longitudinal waves and one quasi-transverse wave in the direction of increasing and decreasing values of z. In the case of the incidence of waves normal to the layers, Eq. (10) decomposes into two independent equations. One of these equations determines the wave numbers of effective longitudinal waves of the first and second types, while the second determines the wave number of the transverse wave. It must be emphasized that the longitudinal waves do not propagate independently in a periodically layered saturated porous medium due to their mutual transformation at the interfaces of the layers. Thus, they are described by a single dispersion relation.

The solution of dispersion relation (10) can be found by numerical methods. Calculation of the effective wave number of the first type of longitudinal wave is of the greatest practical interest. Thus, instead of the term "effective longitudinal wave of the first type," we will henceforth simply refer to "effective longitudinal wave." As the first example, we will present results of calculations for the case when the layers differ only in the filler for the pores, while the effective longitudinal wave propagates in the direction normal to the boundaries of the layers. Similar calculations were performed in [10], where a study was made of unidimensional strains of an element of a porous medium containing layers with different properties. Here, the rigorous theory in [1-4] was not used.

The properties of the skeleton in our calculatons corresponded to those presented in [7] (velocities of the longitudinal and transverse waves in the dry skeleton V_p = 1310 m/sec, V_s = 870 m/sec, density of the material of the skeleton ρ_s = 2650 kg/m³, porosity Φ = 30%, layers filled with water and methane, K_{pr} = 1 μm², compression modulus $K = 3.34 \cdot 10^{11}$ dyne/cm²), the thickness of the layers $h_1 = h_2$ = 10 cm, and the added mass was assumed to be zero ($\rho_{12} = 0$). Figure 1 shows the dependence of the velocity of the effective longitudinal wave on frequency. Curve 1 was taken from [10], while curve 2 was obtained from our calculations. The results show that the velocity of the effective longitudinal wave has

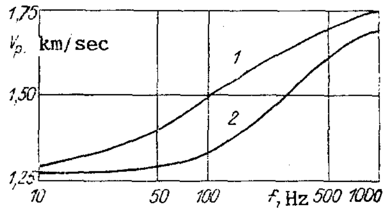

Fig. 1

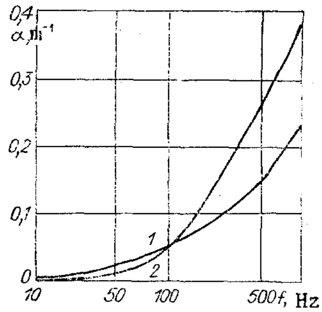

Fig. 2

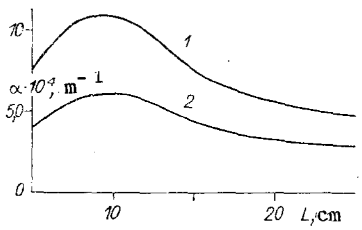

Fig. 3

an appreciable dispersion. Also, such a wave propagating in a laminated medium saturated by layers with gas and liquid has a high attenuation factor. In fact, the latter is more than one order of magnitude greater than the attenuation of the longitudinal wave in each of the layers (Fig. 2). This result can be attributed to the transfer of the energy of longitudinal wave of the first type at the boundaries of the layers into rapidly decaying longitudinal waves of the second type.

Comparison of the results of the calculations (curves 2 in Figs. 1 and 2) with the results in [10] (curves 1 in Figs. 1 and 2) shows that the maximum discrepancy between them is about 30%. Such a deviation can apparently be explained by the fact that the authors of [10] unjustifiably ignored the motion of the skeleton in the calculation of filtrational flows of the fluid at the interfaces of the layers. In the Frenkel–Biot theory used in the present study, the effect of vibrations of the skeleton on filtration of the fluid in the pores is considered automatically.

As the second example, we will examine the case of alternating layers differing in porosity and permeability and saturated with the same fluid. Calculations were performed for a medium with ρ_s = 2700 kg/m², V_p = 6400 m/sec, and V_s = 3700 m/sec. This corresponds, for example, to naturally occurring limestone; the pores are filled with water. The elastic constants in the system (1) were calculated by the method in [11]. The completed calculations show that filtrational flow of fluid at the interfaces of the beds has a slight (fractions of a percent) effect on the velocity of the effective longitudinal wave. Thus, this velocity can be determined through the use of the simpler model of an elastic laminated medium.

Figure 3 shows the dependence of the attenuation factor of the effective longitudinal wave on the period of the structure L. Calculations were performed for the case when the longitudinal wave is propagated normally to the boundary of the layers. The frequency of the elastic waves f = 500 Hz, Φ = 5%, and K_{pr} = 0.01 μm². The porosity and permeability of the second layer was 25; 20% and 1; 0.5 μm² (curves 1 and 2). Despite the fact that the layers are thin for the effective longitudinal wave ($|k_1 L| \ll 1$), the attenuation of the wave depends nonmonotonically on L. This can be explained as follows: with an increase in L, an increasing proportion of the energy of longitudinal waves of the second type can be absorbed and not reach the next boundary, which leads to an increase in attentuation (left side of the graph). With a further increase in L, the degree of attentuation is connected with a decrease in the number of boundaries at which the energy of the first-type longitudinal waves is dissipated by formation of second-type waves.

Thus, the propagation of elastic waves in a periodically laminated, saturated, porous medium has several important features which can be linked to the motion of the fluid relative to the skeleton at the boundaries between the layers. These features cannot be accounted for in the theory of visco-elastic media. They can be best accounted for within the framework of the Frenkel—Biot model.

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PROPAGATION OF COMPRESSION WAVES IN A POROUS FLUID-SATURATED MEDIUM

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Theoretical analysis of the propagation of compression waves in porous media saturated by a fluid [1-3] has shown that the main mechanism determining the evolution of the waves is interphase friction at the boundary of the fluid and the solid skeleton. It was found in [4-7] that one longitudinal wave is propagated in saturated porous media, while [6] presented test data on the decay of high-frequency acoustic waves which were generalized well by calculations performed in accordance with [1]. The authors of [8, 9], examining ultrasonic waves in consolidated porous media, were the first to experimentally detect the existence of two types of longitudinal waves — "fast" and "slow." The goal of the present study is to obtain experimental data on the dynamics of a compression wave in porous media saturated with fluid within a broad range of parameters of the waves and medium. We also want to generalize this data on the basis of calculations performed in accordance with well-known models.

Ignoring convective terms for the liquid and solid phases, the system of equations for the strains of the solid skeleton e_1 and the fluid e_2 in longitudinal waves has the following form in the unidimensional case [1, 10]

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